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# Stock and Structured Warrant Portfolio Optimization Using Black-Litterman Model and Binomial Method

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**Abstract**

*In recent years, the number of Indonesian investors has rapidly increased during the COVID-19 pandemic which happened all around the world. There have been a massive number of influencers in social media who were promoting investment. Although stocks and warrants are interesting choices, mutual funds still become the main ones for beginners. Therefore, this research focuses on the development of a stock portfolio model using the Black-Litterman method which involves the investor's views towards the stock returns. The research refers to one of the largest equity funds in Indonesia, that is Sucorinvest Equity Fund, by using the top ten of its stocks that are majority in the fund (as of April 28, 2023). Furthermore, this research also constructs a structured warrant portfolio, but it is separated from the initially constructed stock portfolio. Structured warrants could be an appropriate choice for low-budget investors. It was newly introduced in Indonesia in September 2022 so it is interesting to be observed. Based on the results and the implemented assumptions, the return obtained from the stock portfolio is superior to the observed fund's return. Meanwhile, call structured warrant portfolio using the existing product in the market yields a negative return, because the exercise price and warrant offered price were too high. Thus, structured warrants could be considered overpriced at the moment, so the chance of obtaining profit is extremely small. Due to its similar properties to call and put options, we propose the warrant pricing and use it in simulations, so in the future, structured warrants may become an attractive instrument for the investors.*

**Keywords:** *Black-Litterman, Stocks Portfolio, Mutual Funds, Structured Warrant, Call and Put, Option Pricing.*

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## 1 Introduction

The COVID-19 pandemic was one of the tragedies that resulted in a financial crisis and change in people's lifestyles. On the other hand, the Indonesian capital market has become a more popular alternative source of income. At the end of 2019, there were around 2.48 million Indonesian investors, and a year after it is increasing significantly to 3.88 million [6]. Unfortunately, most new investors did not understand how to invest their money properly. Consequently, most of them run into huge losses and they might fall into depression. One of the most popular investment instruments is a mutual fund, i.e., a form of investment, consisting of various assets such as stocks, bonds, or others. Thus, it shows great diversification which minimizes the risk and stabilizes return.

Mutual funds are managed by an investment manager who carries greater risk compared to an individual investor because the managed fund is extremely large. Therefore, it is needed to build portfolios using credible quantitative methods. In this research, we are using Markowitz with Black-Litterman allocation in constructing a stocks portfolio, which views returns are predicted using the LSTM model. With this procedure, we expect the constructed portfolio to result in higher and more stable returns. Markowitz's method is a fundamental basis for modern portfolio theory where the optimal portfolio construction is dependent on the mean and variance of return of each asset (based on historical data). Recently, there has been some research about the improvement of the Markowitz model such as [11], collaboration of Markowitz, and Technical Analysis [14]. Besides that, there are also researches using the Black-Litterman method such as combining it with Copula [13] to incorporate tail dependency in portfolio optimization, time-varying Black-Litterman in portfolio optimization [15], application of Black-Litterman method in active portfolio management [16]. This research is a development of [18] where we are using the LSTM model for views return and options are substituted with structured warrants.

Most conventional portfolios only consist of stock and a risk-free asset. They may consist of other types of assets such as options, warrants, bonds, futures, or others. In September 2022, the Indonesia Stock Exchange introduced structured warrant that gives the buyer rights to sell or buy the asset that has been written on the contract. Although options and structured warrants are quite different, the pricing method could be similar due to their properties. Some research about option pricing using adaptive differential evolution and option portfolio construction are [7] and [20].

Similar to options, there are two kinds of structured warrants; call and put warrants where a call warrant gives the right to buy the underlying asset while a put warrant gives the right to buy the underlying asset. Different from options, the execution of a warrant gives the gain from the strike price-asset price difference without buying or selling the underlying asset. Investors can hold warrants as a hedging tool because the movement of the stock price is not always the same as expected. These warrants could protect investors from huge losses when the stock price goes either extremely low or high. Meanwhile, the movement of the stock price is affected by various factors and not all of them can be considered. According to its purpose, we can minimize the loss or even obtain profit from the financial market by taking advantage of the appropriate type of structured warrant.

All these reasons make research on a mutual fund in Indonesia and the utilization of structured warrants very interesting. Therefore, we construct a portfolio based on the same stocks in the mutual fund and structured warrants that are sold in the Indonesian stock market. In this research, we develop active portfolios using the Markowitz method with Black-Litterman allocation and implement them into 2022-2023 data. Using a dynamic portfolio approach, the constructed portfolio will be updated or adjusted in various frequencies for a year; once, twice, four times, or twelve times in a year. Another portfolio containing call and put structured warrants is also constructed. Then the obtained returns from every scheme are compared, especially for the stocks portfolio, we compare it to the mutual fund that we refer to.

The contents of this paper are arranged as follows. The next section shows the basic theories about stock portfolios, structured warrant pricing, and asset allocation. After that, the implementation of the provided methods using real historical data and predictive models, then numerical results are explained. In the end, we have conclusions.

## 2 Portfolio of Stock and Structured Warrant

Suppose there is a portfolio consisting of $N$ assets whose weights are $x \in \mathbb{R}^N$, the expected return vector is $\boldsymbol{E(r)} \in \mathbb{R}^N$, and the variance-covariance matrix is $\boldsymbol{\Sigma} \in \mathbb{R}^{N\times N}$.

**Definition.** The portfolio expected return is calculated with the following equation.

$$E(r_P) = \sum_{i=1}^{N} x_i E(r_i) = \boldsymbol{x}^T \boldsymbol{E(r)}. \tag{1}$$

**Definition.** The portfolio variance is calculated with the following equation.

$$\begin{aligned} Var(r_P) &= \sum_{i=1}^{N} x_i^2 Var(r_i) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} x_i x_j Cov(r_i, r_j) \\ &= \boldsymbol{x}^T \boldsymbol{\Sigma} \boldsymbol{x}. \end{aligned} \tag{2}$$

An efficient portfolio is a portfolio of risk assets that generate minimum variance given an expected return or a portfolio that generates maximum expected return given the variance. Generally, there are three kinds of efficient portfolios, i.e., variance efficient, expected return efficient, and parametric efficient.

**Definition.** Mathematically, variance efficient portfolio can be written as an optimization problem that optimizes:

$$\min Var(r_P) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \boldsymbol{x}^T \boldsymbol{\Sigma} \boldsymbol{x}, \tag{3}$$

subject to:

$$\sum_{i=1}^{N} x_i E(r_i) = \mu_P = E(r_P), \qquad \sum_{i=1}^{N} x_i = 1.$$

**Definition.** An expected return efficient portfolio can be written as an optimization problem that optimizes:

$$\max E(r_P) = \sum_{i=1}^{N} x_i E(r_i) = \boldsymbol{\mu}^T \boldsymbol{x}, \tag{4}$$

subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \boldsymbol{x}^T \boldsymbol{\Sigma} \boldsymbol{x} = {\sigma_P}^2, \qquad \sum_{i=1}^{N} x_i = 1.$$

**Definition.** A parametric efficient portfolio can be written as an optimization problem that optimizes:

$$\min \left( -t\mu_P + \frac{1}{2} Var(r_P) \right) = -t\boldsymbol{\mu}^T \boldsymbol{x} + \frac{1}{2} \boldsymbol{x}^T \boldsymbol{\Sigma} \boldsymbol{x}, \tag{5}$$

subject to:

$$t \geq 0, \qquad \sum_{i=1}^{N} x_i = 1.$$

The stock optimization methods being used in this research are the Markowitz and Black-Litterman allocation methods. The main objective of the Markowitz method is to maximize the Sharpe ratio.

**Definition.** Sharpe ratio is the expected portfolio excess return $(E(r_P) - r_f)$ divided by its risk $\sigma_P = \sqrt{Var(r_P)}$. The optimization problem to be solved is as follows.

$$\max \theta = \frac{E(r_P) - r_f}{\sigma_P} \tag{6}$$

subject to:

$$\sum_{i=1}^{N} x_i = 1.$$

Later, there will be an additional constraint function used in this research that prohibits short selling i.e., $x_i \geq 0,\ i = 1, 2, \ldots, N$.

## 2.1 Black-Litterman Method

Black-Litterman method [8, 9] is an asset allocation model based on the Bayesian approach which combines prior estimates of returns with views on certain assets [3, 4, 12]. The result of this model is a posterior return that generates a more stable portfolio compared to a mean-historical return portfolio. Its formula represents a weighted average of prior estimation and views, where the weighting is determined by the confidence in the views and tuning parameter $\tau$. Prior estimation could be obtained from historical data, meanwhile, views are from investors' opinions. But in this research, we are using the LSTM model to generate the views by predicting stocks' return.

First, we calculate the prior return of each asset using the market-implied return model. Because every asset in the market portfolio contributes a certain level of risk and investors must be compensated for that risk, we can attribute to each asset an expected compensation, i.e., the prior estimate of returns. This compensation is quantified by the market-implied risk premium, it is the market excess return $(R_M - r_f)$ divided by its variance $(\sigma^2)$. The prior return vector $(\mathbf{\Pi})$ could be obtained by formula:

$$\mathbf{\Pi} = \delta \mathbf{\Sigma} \boldsymbol{w_{mkt}}, \qquad \delta = \frac{R_M - r_f}{\sigma_M^2}, \tag{7}$$

where $\mathbf{\Sigma}$ is prior covariance matrix from historical data and $\boldsymbol{w_{mkt}}$ is the weight vector of assets. The next step is determining views return using LSTM, let $\boldsymbol{Q} = (\hat{r_i})$ be $N \times 1$ views return vector. In Black-Litterman model, one can provide either absolute or relative views. Absolute views focus on the return of each asset itself; meanwhile, relative views compare asset returns one to another. As we use LSTM to predict each asset return then we are only using absolute views. After that, we determine picking matrix $\boldsymbol{P}$ and uncertainty matrix $\mathbf{\Omega}$. The picking matrix maps the views to the asset universe meanwhile uncertainty matrix represents the confidence level for the model. In this research, picking matrix $\boldsymbol{P}$ is an identity matrix since all views are absolute. Tuning constant $\tau$ is involved in the formulation, by the rule of thumb we can take $\tau = 0.05$, so the uncertainty matrix can be calculated with the following equation:

$$\mathbf{\Omega} = \tau(\boldsymbol{P}\mathbf{\Sigma}\boldsymbol{P}^T). \tag{8}$$

Finally, we can determine the posterior expected return and posterior covariance matrix, they serve as input for the Markowitz model instead of the prior from historical data. Both could be obtained by the formula:

$$\boldsymbol{E(R)} = [\boldsymbol{W_1} + \boldsymbol{W_2}\boldsymbol{P}]^{-1}[\boldsymbol{W_1}\mathbf{\Pi} + \boldsymbol{W_2}\boldsymbol{Q}], \tag{9}$$

$$\hat{\mathbf{\Sigma}} = \mathbf{\Sigma} + [\boldsymbol{W_1} + \boldsymbol{W_2}\boldsymbol{P}]^{-1} \tag{10}$$

where

$$\boldsymbol{W_1} = (\tau\mathbf{\Sigma})^{-1}, \qquad \boldsymbol{W_2} = \boldsymbol{P}^T\mathbf{\Omega}^{-1}.$$

If there is a short selling restriction, there will be an additional constraint $x_i \geq 0,\ i = 1, 2, \ldots, N$.

## 2.2 Long Short-Term Memory (LSTM)

Long Short-Term Memory is a modification of Recurrent Neural Network (RNN) which is capable of learning long-term dependencies. LSTM has cell states that serve as long-term memory and hidden states for short-term memory. Three gates in LSTM process the information from data, i.e., the forget gate that deletes irrelevant information, the input gate that adds relevant information from new data, and the output gate that determines the output for the model. We use LSTM to predict returns which serve as views. Each stock return for the last 20 days serves as a predictor that needs to be transformed by a min-max scaler. Next, the model is trained with epochs 10 and batch size 16 to predict returns for the next 10 days. The average of these predictive returns will be the views for our Black-Litterman model.

## 2.3 Structured Warrant

Structured warrant is a derivative that gives the holder the right to buy/sell (call or put) shares on a specific date (exercise date) at a predetermined price (exercise price). It is similar to option but there are some differences, especially in the settlement method and its valuation. When an option is exercised, it means we buy/sell the real stocks. Meanwhile, a structured warrant is automatically exercised, which means the writer only pays the settlement in cash. We can calculate cash settlement using the formulas below.

$$St_{call} = n_{call} \times \max\left(\frac{St_{price} - K_{call}}{Ex_{ratio}}, 0\right), \tag{11}$$

$$St_{put} = n_{put} \times \max\left(\frac{St_{price} - K_{put}}{Ex_{ratio}}, 0\right). \tag{12}$$

For $k = \{call, put\}$, $St_k$ is cash settlement for the structured warrant, $n_k$ is number of shares, $K_k$ is exercise price, $St_{price}$ is settlement price which is average of five days price of the underlying stock before it is exercised, and $Ex_{ratio}$ is conversion ratio warrant to stock. We can distinguish each variable for the call or put warrant by notes in subscript. Since structured warrant is a new instrument that began to be introduced in Indonesia in September 2022, there is no research about its previous valuation yet. Therefore we try the option pricing approach to evaluate its fair price, i.e., using the Binomial CRR (Cox-Ross-Rubinstein) method. This method assumes stock prices only move upward or downward where upward factor $u = e^{\sigma\sqrt{dt}}$ which probability $p = \frac{e^{r_f \Delta t} - d}{u - d}$, downward factor $d = \frac{1}{u}$ which probability $q = 1 - p$. We determine the fair value of structured warrant by using backward recursion, letting $V_{ij}$ denote the fair value of the warrant at period $i$ when the stock price moves upward $j$ times for

$i = 1, 2, \ldots, T-1$, then we have:

$$V_{ij} = e^{-r_f \Delta t} \left(pV_{i+1,j+1} + (1-p)V_{i+1,j}\right). \quad (13)$$

Here we take $\Delta t = \frac{1}{252}$ (in years) as we are using the risk-free rate in a year. The recursion starts from $i = T-1$ where $T$ is the number of days from the starting date to the exercise date. At the exercise date, we can express fair value as cash settlement. Let $V_{Tj} = C_{Tj}$ for call warrant and $V_{Tj} = P_{Tj}$ for put warrant, then we can formulate both as shown below.

$$C_{Tj} = \max\left(\frac{St_{price} - K_{call}}{Ex_{ratio}}, 0\right), \quad (14)$$

$$P_{Tj} = \max\left(\frac{K_{put} - St_{price}}{Ex_{ratio}}, 0\right). \quad (15)$$

### 2.4 Structured Warrant Optimization

As a structured warrant is a derivative, applying a stock optimization method such as Markowitz is not a suitable choice. Thus, we try another approach to allocate a warrant portfolio using a calculated fair price based on binomial CRR. There are 4 methods that we apply in this research, i.e., raw return, scaled return, uniform, and price ratio. Raw return and scaled return utilize the expected return of warrant fair price, the one which return is greater, its portion is higher. Let $x_i^d$, $d = \{1, 2, 3, 4\}$ denote the portion of warrant $i$ which refers to the method being calculated; raw return, scaled return, uniform, and price ratio respectively. then the allocation formula with raw return and scaled return are given as (16) and (17).

$$x_i^1 = \frac{\max(r_i, 0)}{\sum_{i=1}^{N} \max(r_i, 0}, \quad (16)$$

$$x_i^2 = \frac{r_i^s}{\sum_{i=1}^{N} r_i^s}, \quad r_i^s = \frac{r_i - \min_i(r_i)}{\max_i(r_i) - \min_i(r_i)}. \quad (17)$$

Another method is uniform, which means we allocate the money for all warrants at the same proportion if there are $N$ warrants then $x_i^3 = \frac{1}{N}$ for all $i = 1, 2, \ldots, N$. The last method is price ratio allocation where we need to calculate the ratio of predicted selling price (fair price) to buying price at that moment. The higher ratio means a higher expected return so the portion will be higher. Then we transform the ratio using a min-max scaler, so these scaled ratios determine warrant portions. Let $t_i$ be the price ratio then we get the formula shown below.

$$x_i^4 = \frac{t_i^s}{\sum_{i=1}^{N} t_i^s}, \quad t_i^s = \frac{t_i - \min_i(t_i)}{\max_i(t_i) - \min_i(t_i)} \quad (18)$$

## 3 Implementation

In establishing stock and structured warrant portfolios, the stocks being used were intended chosen from the top 10 largest proportion stocks in the mutual fund of Sucorinvest Equity Fund as of April 2023 [17].

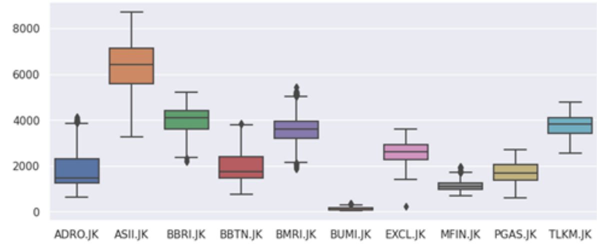


Figure 1: Box plot of closing stock prices (in IDR) for January 2018 – May 2023.

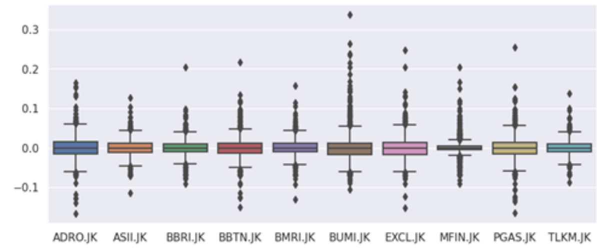


Figure 2: Box plot of daily stock returns for January 2018 – May 2023.

Ten stocks that are included in IDX are ADRO, ASII, BBRI, BBTN, BMRI, BUMI, EXCL, MFIN, PGAS, and TLKM. These stocks are grouped into several sectors; ADRO, BUMI, and PGAS are stocks in the energy sector; ASII is a stock in the automotive-industrial sector; BBRI, BBTN, BMRI, and MFIN are financial sector stocks; EXCL and TLKM are stocks in infrastructure (communication services) sector. The risk-free asset was assumed to be Batavia Dana Kas Maxima [2] because it is a money market fund in which the risk is very low (Batavia Dana Kas Maxima has not resulted in a negative return since it was listed). To simplify the model, we assumed risk risk-free rate is 4%, i.e., rounding down of average annual return of Batavia Dana Kas Maxima for the past 5 years (4.02%). Meanwhile, for structured warrants, we chose 5 call warrants that will be exercised earliest (May 5, 2023), i.e., ANTMDRCK3A, BBCADRCK3A, BMRIDRCK3A, MDKADRCK3A, and PGASDRCK3A [5], where the writer of these warrants is RHB Sekuritas. In simulation, we also considered put warrant but due to the unavailability of it in the real market, we created the fiction one. The parameters from or put warrant were taken the same to call warrant. In trading of put warrant was done by taking a fair price; both for buying and selling. Simulations for the stock portfolio and structured warrant portfolio were done separately. To see the initial condition of the overall capital market, the author observed the movement of the stock price from January 2018 to May 2023, showing that five stocks (ADRO, BBRI, BMRI, MFIN, and TLKM) had increasing prices, two stocks (BBTN and BUMI) had decreasing prices, and the other three (ASII, EXCL, and PGAS) had a sideways trend.

### 3.1 Descriptive Statistics of Stock Prices and Returns

First, we analyzed stock prices and returns based on historical data to get a prior idea about their characteristics. Box plots of stock prices and returns will be depicted as Figure 1 and Figure 2 which show quartiles and whiskers (upper and lower).

As shown in Figure 1, we may notice that ASII is the most expensive whereas the others have average prices of less than IDR 5,000 per share. Based on Figure 2, the medians of all stocks' daily returns are nearly 0.00%.

### 3.2 Assumptions in Constructing Portfolio

As call warrants were listed on November 10, 2022, and their exercise dates are the same, i.e., May 10, 2023, we construct a stock portfolio for one year from May 10, 2022, to May 10, 2023, for structured warrant portfolio, we construct it for 6 months period from November 10, 2022, to May 10, 2023. Therefore, we are using historical data from January 1, 2018, to May 9, 2022. The stock portfolio was updated at various frequencies, i.e., annually, biannually, quarterly, and monthly; at the end, we will compare their performances. The structured warrant portfolio was divided into call and put; both were updated monthly. There were some assumptions that we set:

1. No short selling,
2. Risk-free rate is 4% per year,
3. Buying fee is 0.15% and selling fee is 0.25%,
4. Transaction is done in minimum buying unit (100 shares),
5. The Initial balance for the stock portfolio is IDR 1,000,000,000 and for the warrants portfolio is IDR 100,000,000 (each call and put is IDR 100,000,000),
6. Net worth (net asset value after deducting fee when all assets were sold) is the balance allocated for the next period.

### 3.3 Optimal Portfolio of Stocks

Portfolio construction was made dynamically which means there would be adjustments according to each frequency. For schemes with multiple portfolio constructions in a year, the buying and selling activities were held at every $10^{\text{th}}$, in case that date was a holiday, the transaction was done at the latest date before $10^{\text{th}}$. Due to data availability, we determined the proportion of each stock on the date before adjustment was done. Adjusting for the new weights obtained for the next period was done efficiently, which means we only bought or sold at the exact difference of proportion; thus, we paid the fees as little as possible.

We were using the Black-Litterman method in which opinions on the returns were obtained through LSTM model using historical data. First, the prior return was determined using market-implied return model. We considered the risk contribution of each stock in the portfolio then determined the fair return using IHSG (Indeks Harga Saham Gabungan), or IDX Composite as a benchmark, from January 2018 to the period before portfolio construction. Next, we determined views return using LSTM, where it came from the average of predicted 10-day return. Combining prior return and views return using formula (9), we got posterior return. Table 1 shows examples of prior, views, and posterior returns in the first period (May 2022).

Table 1: Prior, views, and posterior stocks' return.

| Stock | Prior Return | Views Return | Posterior Return |
|---|---|---|---|
| ADRO | 0.051986 | 0.824180 | 0.117245 |
| ASII | 0.053005 | 0.494014 | 0.092231 |
| BBRI | 0.067010 | 1.455009 | 0.109300 |
| BBTN | 0.060906 | -0.091554 | 0.108582 |
| BMRI | 0.063925 | 1.317200 | 0.107488 |
| BUMI | 0.037499 | 0.645723 | 0.067763 |
| EXCL | 0.052041 | 0.528977 | 0.094305 |
| MFIN | 0.026641 | 0.998330 | 0.063411 |
| PGAS | 0.058893 | 2.110187 | 0.144058 |
| TLKM | 0.049982 | -0.293635 | 0.064570 |

Besides the expected return, Black-Litterman model also results in a posterior covariance matrix which is important in determining a low-risk portfolio. In computing the uncertainty matrix, we need a confidence level for each stock, in this case, we provide a qualitative but objective opinion. As the views return from the LSTM model is quite extreme (either low or high) then it is reasonable to take a low confidence level. For more stable banking stocks such as BBRI, BBTN, BMRI, we took the lowest confidence level as we expected the return would not be that extreme. Meanwhile, for trending stocks (ADRO, ASII) and a small-cap stock (MFIN), we provided the highest confidence level as it may be more possible to reach those levels of return. Table 2 shows the assumed confidence level for each stock, we used the same level for each period.

Then we determined the posterior covariance matrix using the formula (8) and (10), this matrix is included in the portfolio variance formula (2). After getting the expected return of each stock and its risks (posterior covariance matrix), we obtained an optimal portfolio that maximizes Sharpe Ratio (6) as Table 3.

After that, we constructed a stock portfolio based on each portion, and the transactions were done in minimum buying units. In case there was a remaining balance (insufficient balance for trading), we invested it in the risk-free asset that gives a rate of return of 4% per year. The simulation was done in all schemes using the Markowitz method, Table 4 shows the results in ending balance, gain in IDR, and annual return. Here gain was calculated as the ending balance minus the initial balance.

Some discussions based on the results of the optimal stock portfolio in Table 4 are the following.

1. Returns obtained from the simulated portfolio show decent performances, where they are much higher than the risk-free rate (4%).

Table 2: Prior, views, and posterior stocks' return.

| Stock | Confidence Level |
|---|---|
| ADRO | 0.05 |
| ASII | 0.05 |
| BBRI | 0.01 |
| BBTN | 0.01 |
| BMRI | 0.01 |
| BUMI | 0.02 |
| EXCL | 0.03 |
| MFIN | 0.05 |
| PGAS | 0.03 |
| TLKM | 0.03 |

Table 3: Optimal stock portfolio (May 2022).

| Stock | Portion |
|---|---|
| ADRO | 14.150% |
| ASII | 9.451% |
| BBRI | 1.305% |
| BBTN | 11.332% |
| BMRI | 12.571% |
| BUMI | 3.141% |
| EXCL | 8.581% |
| MFIN | 5.244% |
| PGAS | 19.899% |
| TLKM | 2.326% |

2. Returns obtained from simulation are dependent on adjustment frequency; the more often adjustment is done, the higher expected return will be obtained. It may happen because investors could avoid bigger losses by doing portfolio adjustments.
3. Portfolio construction using Markowitz with Black-Litterman shows a stable portfolio, it is indicated by net worth growing smoothly as shown in Figure 3.
4. Simulated portfolios show better performance, i.e., the returns were higher than the actual return of the mutual funds reported by Sucor Asset Management [17], which was -6.20%.

### 3.4 Portfolio of Call and Put Structured Warrants

Due to the non-existence of put structured warrants in Indonesia until this research was done, we experimentally designed the fiction put warrants that were similar to offered call warrants. By using historical data, we could obtain the upward factor and its probability for each stock. Then we calculated a fair price for each warrant using the Binomial CRR method, especially for put warrants, the initial fair prices functioned as offered prices. Table 5 shows the offering price for each warrant (call and put), which $7^{th}$ letter shows warrant type (C for call and P for put).

Fair prices were determined every month, following the adjustment period. Figure 4 and Figure 5 show the movement of warrant fair prices since they were listed till the last trading date.

According to the figures above, we may spot that the fair prices are decreasing as it was close to the exercise date. Theoretically, call warrant and put warrant prices contrast each other because, at the exercise date, only one of them would be exercised. As the figures show when it was closer to the exercise date, one of them would be approaching zero. For example, on May 5, 2023 (month 6 in graphs) which was close to the exercise date (May 10, 2023), MKDADRCK3A and PGASDRCK3A had fair prices equal to zero, but the put warrants (MDKADRPK3A and PGASDRPK3A) had fair prices IDR 125.81 and IDR 172.96 respectively.

After we get the fair prices for each period, we will construct a structured warrant portfolio with monthly adjustments starting on November 10, 2022. The last adjustment was held on April 5, 2023, then on May 5, 2023 (the last trading date), we did not buy any warrants, instead, we just decided whether to sell them or hold them till the exercise date. So, on May 5, 2023, we would calculate the predicted settlement value when exercising warrants, if the predicted settlement was higher than the trading prices then we would sell them immediately otherwise we would hold them for exercise.

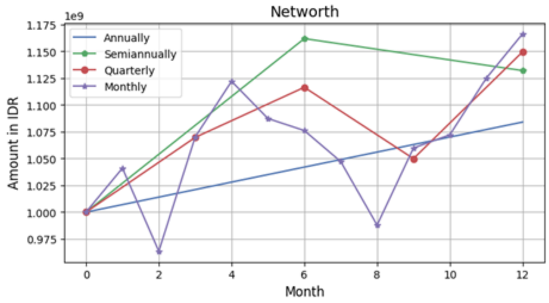


Figure 3: Net worth of stocks portfolio for each scheme.

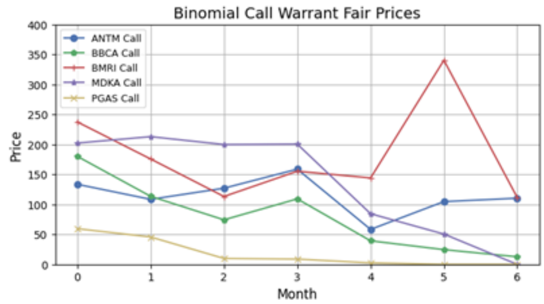


Figure 4: Call warrant fair prices.

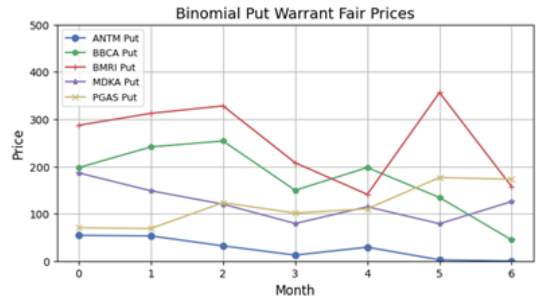


Figure 5: Put warrant fair prices.

Table 4: Optimal stock portfolio simulation results.

| Adjustment Frequency | Ending Balance | Gain | Annual Return |
|---|---|---|---|
| Annually | IDR 1,084,102,346 | IDR 84,102,346 | 8.4102% |
| Biannually | IDR 1,132,147,024 | IDR 132,147,024 | 13.2147% |
| Quarterly | IDR 1,149,678,892 | IDR 149,678,892 | 14.9679% |
| Monthly | IDR 1,166,505,435 | IDR 166,505,435 | 16.6505% |
| Sucorinvest Equity Fund | IDR 938,000,000 | -IDR 62,000,000 | -6.20% |

Table 5: Optimal stock portfolio simulation results.

| Structured Warrant | Offering Price | Structured Warrant | Offering Price |
|---|---|---|---|
| ANTMDRCK3A | IDR 196 | ANTMDRPK3A | IDR 54.35 |
| BBCADRCK3A | IDR 550 | BBCADRPK3A | IDR 197.55 |
| BMRIDRCK3A | IDR 645 | BMRIDRPK3A | IDR 287.10 |
| MDKADRCK3A | IDR 300 | MDKADRPK3A | IDR 186.42 |
| PGASDRCK3A | IDR 116 | PGASDRPK3A | IDR 70.37 |

Table 6: Optimal structured warrant portfolio (November 2022).

| Warrant | Raw Return | Scaled Return | Uniform | Price Ratio |
|---|---|---|---|---|
| ANTMDRCK3A | 1.6578% | 0.0000% | 20% | 14.7875% |
| BBCADRCK3A | 25.3180% | 25.7986% | 20% | 17.0181% |
| BMRIDRCK3A | 17.5761% | 17.3570% | 20% | 0.0000% |
| MDKADRCK3A | 34.4157% | 35.7186% | 20% | 40.1222% |
| PGASDRCK3A | 21.0325% | 21.1258% | 20% | 28.0722% |
| ANTMDRPK3A | 73.3850% | 82.7523% | 20% | 17.1930% |
| BBCADRPK3A | 3.8458% | 1.0112% | 20% | 39.5764% |
| BMRIDRPK3A | 2.9855% | 0.0000% | 20% | 26.9680% |
| MDKADRPK3A | 16.3347% | 15.6915% | 20% | 0.0000% |
| PGASDRPK3A | 3.4491% | 0.5449% | 20% | 16.2626% |

Table 7: Predicted settlement value for each structured warrant.

| Call Warrant | Settlement | Put Warrant | Settlement |
|---|---|---|---|
| ANTMDRCK3A | IDR 91.40 | ANTMDRPK3A | IDR 0.00 |
| BBCADRCK3A | IDR 0.00 | BBCADRPK3A | IDR 38.05 |
| BMRIDRCK3A | IDR 0.00 | BMRIDRPK3A | IDR 43.00 |
| MDKADRCK3A | IDR 0.00 | MDKADRPK3A | IDR 179.89 |
| PGASDRCK3A | IDR 0.00 | PGASDRPK3A | IDR 174.29 |

Table 8: Structured warrant selling prices on May 5, 2023.

| Call Warrant | Sell Price | Put Warrant | Sell Price |
|---|---|---|---|
| ANTMDRCK3A | IDR 108 | ANTMDRPK3A | IDR 0 |
| BBCADRCK3A | IDR 86 | BBCADRPK3A | IDR 44 |
| BMRIDRCK3A | IDR 122 | BMRIDRPK3A | IDR 157 |
| MDKADRCK3A | IDR 2 | MDKADRPK3A | IDR 125 |
| PGASDRCK3A | IDR 3 | PGASDRPK3A | IDR 172 |

Using the Binomial CRR method for valuation and four allocation methods, we could obtain each warrant proportion in the portfolio. Expected returns and price ratios were used to determine the proportions as formula (16), (17), and (18). The proportion of each warrant for the first period (November 2022) is shown in Table 6.

The portfolios were adjusted every month until April 2023, then on May 5, 2023, we predicted the settlement value for each warrant using the Binomial CRR method. Table 7 shows the predicted settlement value per share for each structured warrant, where if the call warrant has a non-zero predicted settlement, then the put warrant predicted settlement is zero, and vice versa.

Table 8 shows the selling price for each warrant on May 5, 2023, for the call warrant we were using real price in the market, meanwhile for the put warrant we were using fair price at that date. We compared these prices to predicted settlements to decide whether we hold those warrants or just sell them.

Table 9 shows real settlement for each warrant, where these were obtained on May 10, 2023 (exercise date). If we compare these values to the predicted ones, they are not significantly different except for MDKADRPK3A. Anyway, the decision to sell ANTMDRCK3A, BBCADRCK3A, BMRIDRCK3A, MDKADRCK3A, PGASDRCK3A, BBCADRPK3A, BMRIDRPK3A and to hold MDKADRPK3A was accurate. Although there was inaccuracy in holding PGASDRPK3A (real settlement is lower than selling price), it was tolerable due to the slight difference between predicted settlement, selling price, and real settlement.

After the simulation was done for each method until the exercise date (May 10, 2023), we could calculate the ending balance of structured warrant portfolios where all schemes were using the same initial balance, i.e., IDR 100,000,000. To compare the performances, we could also calculate the gain and return as shown in Table 10 for call warrant portfolios and Table 11 for put warrant portfolios.

Some discussions based on the results of the optimal structured warrant portfolio are the following.

Table 9: Real settlement value for each structured warrant.

| Call Warrant | Real Settlement | Put Warrant | Real Settlement |
|---|---|---|---|
| ANTMDRCK3A | IDR 98.00 | ANTMDRPK3A | IDR 0.00 |
| BBCADRCK3A | IDR 0.00 | BBCADRPK3A | IDR 43.33 |
| BMRIDRCK3A | IDR 0.00 | BMRIDRPK3A | IDR 66.67 |
| MDKADRCK3A | IDR 0.00 | MDKADRPK3A | IDR 212.00 |
| PGASDRCK3A | IDR 0.00 | PGASDRPK3A | IDR 170.00 |

Table 10: Ending balance, gain, and return of call structured warrant portfolios.

| Call Warrant | Raw Return | Scaled Return | Uniform | Price Ratio |
|---|---|---|---|---|
| **Balance** | IDR 3,314,343 | IDR 3,248,316 | IDR 14,291,206 | IDR 43,649,036 |
| **Gain** | -IDR 96,685,657 | -IDR 96,751,684 | -IDR 85,708,794 | -IDR 56,350,064 |
| **Return** | -96.6857% | -96.7517% | -85.7088% | -56.3510% |

Table 11: Ending balance, gain, and return of put structured warrant portfolios.

| Put Warrant | Raw Return | Scaled Return | Uniform | Price Ratio |
|---|---|---|---|---|
| **Balance** | IDR 4,624,899 | IDR 5,501,177 | IDR 91,106,297 | IDR 615,846,109 |
| **Gain** | -IDR 95,375,101 | -IDR 94,498,823 | -IDR 8,893,703 | IDR 515,846,109 |
| **Return** | -95.3751% | -94.4988% | -8.8937% | 515.8461% |

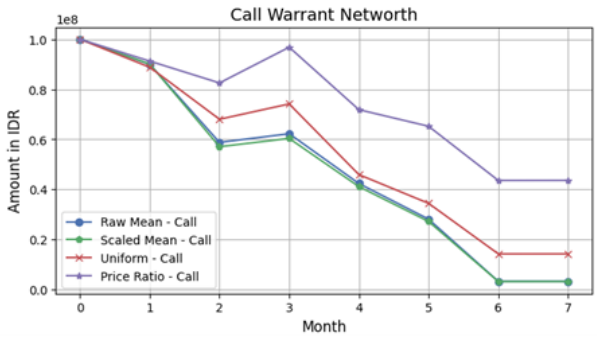


Figure 6: Net worth of call structured warrant portfolio for each scheme.

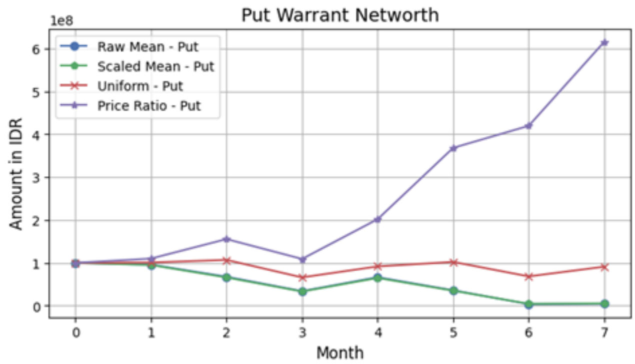


Figure 7: Net worth of put structured warrant portfolio for each scheme.

1. Returns obtained from call warrant portfolios are all-negative, this makes sense because most of the underlying stock prices were decreasing and were less than the exercise price. Only one warrant could be exercised, i.e., ANTMDRCK3A with a settlement value of IDR 98, which was much lower than its offering price (IDR 196). Besides that, the exercise prices were too high, so the probability of exercising the warrants was very low. Thus, we could state that the offering prices were too high for such warrants. Net worth movement of call warrant portfolios is shown in Figure 6.

2. Returns obtained from put warrant portfolios are majorly negative except the one using the Price Ratio method. It means the Price Ratio method is dominating among other methods as it is the only one that results in a very high return (515.8461%). Anyway, this method also shows much better performance in call warrant portfolios when others resulted in loss of more than 80%, this method only resulted in 56% loss. Net worth movement of put warrant portfolios is shown in Figure 7.

3. Put warrant portfolio could produce such high return as we were using fair prices though real prices in the market may be different. So, with the assumption that all trading prices are fair, a high return can be obtained.

4. Structured warrant may be an alternative for investment, especially for a special portfolio with a high expected return target. In addition, this could be interesting when put warrant exists in the Indonesian market and it may be broadly traded in the future.

## 4 Conclusion

Having researched this topic, we have some conclusions. Constructed stock portfolios using Markowitz with Black-Litterman model showed decent performances where the annual returns were 8.41% for annual adjustment, 13.21% for biannual adjustment, 14.97% for quarterly adjustment, and 16.65% for monthly adjustment. These returns are much higher than the assumed risk-free rate return of 4%. Adjustment frequency affects portfolio expected return, the more often adjustment is done, the higher expected return obtained as adjustment prevents bigger losses from happening. In the simulation, returns of constructed portfolios are much higher than the mutual fund return that we referred to, i.e., -6.2%.

The performance of the call structured warrant portfolio was not good enough, even with the best method (Price Ratio) in this research, we got the return was -56.35%. This happened because the exercise prices were too high and most of the underlying stocks did not reach those borders. Meanwhile, put structured warrant portfolio was the opposite, where it showed the possibility of gaining profit as with the best method (Price Ratio), we obtained a return of more than 500%. This happened as we were using fair prices though the reality might be different. Therefore, it is important to choose proper warrants that result in an appropriate return. In the future, structured warrants may become popular in the Indonesian stock market with better performance.

The supplementary material accompanying this article contains the code and data needed are publicly available at Github[1].

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